

Cavity polaritons in the presence of symmetry-breaking disorder: closed-path time formalism

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According to the mean-field theory of Zittartz, when subject to a symmetry-breaking disorder, the order parameter and the energy gap of an excitonic insulator are gradually suppressed up to a critical disorder strength. Recently, Marchetti, Simons, and Littlewood have used a replica trick to investigate the effects of disorder on the condensation of cavity polaritons. Within their nonlinear sigma model, it was found that the saddle-point equations assume the form reported previously by Zittartz in the contest of the symmetry broken excitonic insulator, but with an order parameter, to which both photons and excitons contribute. In this paper, we apply a closed-path time Green's function approach as an alternative to the replica technique to formulate a nonperturbative description of cavity polaritons in the presence of a symmetry-breaking disorder. A field theoretical method is used to derive the Schwinger-Dyson equations for the average photon field and the average single-particle Green's function. In contrast with the nonlinear sigma model and the corresponding saddle-point equations, we obtain that the exact Schwinger-Dyson equations cannot be mapped to the corresponding equations derived by Zittartz. This result not only shows that the theory of Zittartz cannot be applied to the excitons in a disordered quantum well coupled to the cavity photons with only minor modifications, but arises a question about the validity of the replica trick as well.

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I. INTRODUCTION

The phenomenon of the Bose-Einstein condensation (BEC) in atomic gases and superconductors attracts much attention in recent years. Substantial efforts has been recently devoted to BEC of excitons in quantum wells (QW) and microcavities (MC)¹. QW embedded within semiconductor MC have attracted considerable interest^{2,3} due to the following two reasons. Firstly, the recent progress in the growth and manipulation techniques of semiconductor heterostructures allows us to control the coupling between photons and excitons. The QW excitons embedded in semiconductor MC may be found in either weak- or strong-coupling regimes. In what follows we assume the strong coupling regime, where the photon-exciton interaction is larger than the exciton and photon damping rates, and therefore, the normal modes are mixed exciton-photon modes, called cavity polaritons. Secondly, it is expected that the cavity polaritons should have bosonic behavior, and so are candidates for Bose condensation.

Despite the progress made in semiconductor technology, a weak disorder may exist due to the following reasons: the interface roughness, the local thickness fluctuations during crystal-growth processes, randomly distributed impurities, boundary irregularities, and fluctuations of the alloy concentration of the epitaxial layers. The disorder causes additional problems in the theory because all quantities of interest depend on the corresponding random potential, created by the disorder. The actual potential is unknown, but it is not important for the physical properties. Instead, the disorder is considered by means of the probability distribution of the random potential, i.e. one should perform the averaging over all

possible random potentials.

Turning our attention to the theoretical situation, we find that some authors³ have focused on the model, which assumes that the excitons are localized by the disorder, and can be described as two-level oscillators coupled to the light. In this model the original electron-hole-photon Hamiltonian is reduced to the Hamiltonian, which describes the so-called generalized Dicke model.⁴ The model is valid only for describing the very low energy excitonic states, and cannot be applied to a symmetry-breaking disorder, because it assumes that there exists only a symmetry preserving disorder potential, which is strong enough to localize the excitons.

More complicated approach to the problem of excitons (or excitonic polaritons) in weakly disordered semiconductors is based on the assumption that the disorder affects only the center-of-mass motion, but does not affect the exciton internal degrees of freedom (see, e.g., Ref. [5] and references therein). Recently, this idea has been applied to the cavity polaritons.⁶ According to this disorder-affected-center-of-mass-motion (DACMM) approach, the two-particle Schrodinger equation for an isolated exciton separates into two equations: the Wannier equation for the relative motion of the electron-hole pair and the Schrödinger equation for the exciton center-of-mass motion in a random potential. As a result, the two-particle exciton wave function can be factorized, and therefore, the coupling strength of an exciton to light is a random quantity, which depends on the exciton center-of-mass eigenfunctions $\Psi_i(\mathbf{R})$. The next step in the DACMM approach is to use numerical simulations to generate random potentials. Once the energies E_i and eigenfunctions $\Psi_i(\mathbf{R})$ are calculated for a particular random potential, the radiative decay rates, the absorp-

tion (the optical density), or the exciton-photon coupling strengths can easily be evaluated numerically on a grid of a given number of points. There exists a many-body version of DACMM approach,⁷ where instead of numerical simulations, the Green's function of the center of mass of an isolated exciton in the random field is calculated in the coherent potential approximation. It is expected that the factorization of the wave function is justified in the very low density regime. Strictly speaking, the DACMM approach is based on the assumption that the disorder and the interactions (Coulomb and electron-photon interactions) could be treated independently. In other words, the factorization assumption greatly simplifies the problem, but it separates the disorder and the interactions, and therefore, we may expect that the DACMM approach underestimates the influence of the disorder and may lead to incorrect conclusions.

Decades ago, Zittartz⁸ demonstrated that the disorder and the interactions can be treated simultaneously if we perform the averaging over the disorder in the beginning of all calculations. Within this approach, the Green's functions are defined as $< \hat{T}\{...\} >$ and the brackets $< ... >$ denote a thermal average, while \bar{f} means the average of f over the random potential created by the disorder. Zittartz applied the Abrikosov and Gor'kov theory,⁹ developed for the case of superconductors in the presence of a symmetry-breaking disordered potential, to the case of an excitonic insulator in the presence of normal impurities. The theory of Zittartz can be used with only minor modifications to investigate the effects of a symmetry-breaking disorder potential on the two-dimensional excitonic condensate in a high density regime, where the screened Coulomb interaction could be replaced by a contact interaction with a coupling strength g_c . Assuming a Gaussian disorder potential with zero mean, and variance $\overline{V(\mathbf{r})V(\mathbf{r}')} = \Lambda\delta(\mathbf{r}-\mathbf{r}')$, one can obtain the following set of equations for the order parameter Δ at the Fermi surface:

$$\Delta = \frac{g_c}{\beta} \sum_{m=-\infty}^{\infty} \frac{1}{\sqrt{1+u_m^2}}, \quad (1)$$

$$\frac{\omega_m}{\Delta} = u_m \left[1 - \frac{\alpha}{\sqrt{1+u_m^2}} \right].$$

Here $\alpha = 2\Lambda m_{exc}/\Delta$, m_{exc} is the exciton reduced mass, $\omega_m = (2m+1)\pi/\beta$, $\beta = (kT)^{-1}$, where T and k are the temperature and the Boltzmann constant, respectively. The Matsubara summation in (1) must be cutoff at the energy ϵ_0 , which depends on the chemical potential μ . The solution of the above equations shows that: (i) the order parameter and the energy gap are gradually suppressed up to a critical disorder strength; (ii) the suppression of the energy gap is more rapid than that of the order parameter, which means that the existence of a gapless condensed phase is possible.

Generally speaking, the case of cavity polaritons in the presence of a symmetry-breaking disorder is more complicated than the excitonic condensate, because the theory

has to take into account the photonic contributions to all quantities of interest. To the best of our knowledge, there exists only one paper by Marchetti, Simons, and Littlewood¹⁰ (MSL), where a model for cavity polaritons in the high density regime in the presence of a symmetry-breaking disordered potential is proposed, treating the disorder and the interactions simultaneously. MSL have used the so-called replica trick to perform the averaging over the disorder. The replica trick¹¹ is based on the following relationship: $\overline{\ln Z} = \lim_{N \rightarrow 0} [(\overline{Z^N} - 1)/N]$, where Z is the generating functional. Once replicated, MSL have decoupled the arising quartic term in $\overline{Z^N}$ by means of the Hubbard-Stratonovich transformation with the introduction of a matrix field $Q(\mathbf{r}, \omega_m)$, and then, integrating over the fermionic fields the problem is reduced to the so-called nonlinear sigma-model action, previously used to study superconductors with magnetic impurities.¹² The final step in this approach is to draw conclusions by investigating the structure of the saddle-point solution. At the level of the saddle-point approximation the following three statements take place:

(i) while the chemical potential does not exceed the cavity edge mode ω_c , an order parameter $|\Delta| \neq 0$ is developed:

$$|\Delta| = g|\psi| + |\Sigma|. \quad (2)$$

Here, g is the exciton-photon coupling constant, ψ is the average photonic field, $g|\psi|$ and $|\Sigma|$ are the photonic and the excitonic contributions to the order parameter, respectively;

(ii) the excitonic order parameter $|\Sigma|$ and the photonic field $|\psi|$ are not independent quantities because of the following constrain:

$$(\omega_c - \mu)|\psi| = (g/g_c)|\Sigma|; \quad (3)$$

(iii) the saddle-point equations in the high density regime can be mapped to the corresponding set of equations by Zittartz (1), but with the order parameter $|\Delta|$, defined by (2), and with g_c replaced by $g_{eff} = g_c + g^2/(\omega_c - \mu)$. Because of the correspondence between the Zittartz's equations and the saddle-point equations, MSL have concluded that in the low-density regime, where the excitations are mainly excitonic like (only a small fraction of photons contributes to the condensate), the order parameter and the energy gap are gradually suppressed up to a critical strength of the disorder. The suppression of the energy gap is more rapid than that of the order parameter, and therefore, the existence of a gapless condensate is possible. When the density of the excitations is increased, the chemical potential rises (first linearly with the density), and when the chemical potential approaches ω_c , the excitations become photonic like. In other words, the character of the condensate changes from being excitonic to photonic.

The purpose of this paper is to show that all of the above conclusions are drawn only because MSL have performed the averaging over the disorder using the replica trick. To justify our point, we shall treat the

effects of a symmetry-breaking disorder on the cavity polaritons by applying the closed-path time (CPT) (or Keldysh) Green's function technique¹³. The main reason for using this approach is that in the case of a static random potential the Keldysh closed contour in the time direction leads to an automatically disorder independent generating functional. In other words, the CPT approach allows us not only to avoid the need to introduce replicas, but to perform the averaging over the disorder in the beginning of all calculations as well, which is the main requirement when the disorder and the interactions are treated simultaneously. The special form of the Keldysh time contour automatically ensures that the denominator in the representation of the Green functions via functional integrals is equal to unity. The last allows us to derive the exact equations for the average photon field and for the average single-electron Green's function. In the quantum-field theory, these equations are known as the Schwinger-Dyson (SD) equations. In contrast with the saddle-point equation (2), the exact SD equations clearly indicate that in the presence of a symmetry-breaking disorder the photonic contribution to the mass operator is not proportional to the average photon field. We shall see that the results derived by applying the replica trick correspond to the assumption that one can replace the average of the product of two random functions with the product of the corresponding average functions. However, it is known that the average of the product is not the product of the averages, and therefore, the validity of the conclusions based on the replica trick is questionable. The exact result that the photonic contribution to the mass operator is not proportional to the average photon field does not allow us: (i) to map the SD equations to the corresponding Zittartz equations; (ii) to use the Ward identities in order to prove the existence of the Goldstone mode below the critical temperature, and therefore, the question about the existence of a condensate of cavity polaritons in the presence of a symmetry-breaking disorder remains open.

The remainder of the paper is organized as follows. In Sec. II, we discuss the formation of a condensate in the absence of a disorder. This is because we intend to check the validity of the saddle-point approximation by eliminating the effects generated by the replica trick. By applying the Matsubara Green's function method we demonstrate that the polariton spectra can be obtained from the common poles of the photon and the two-particle electron-hole Green's functions. It turns out that the saddle-point equations in the absence of a disorder lead to the same conclusions as those drawn by applying the Matsubara Green's function method. The approach used in Sec. II is very general and, in principle, it is able to treat any density regimes. It also allows us to prove the existence of the Goldstone mode below the critical temperature. Our method provides a set of coupled BCS and Bethe-Salpeter (BS) equations similar to the corresponding equations for

an excitonic condensation.^{14,15} Due to the photonic contribution to the condensate, the BCS and the BS equations are more complicated than the equations reported in our previous paper¹⁵, and it would be a very challenging task to solve them in the case of a low-density limit. Such an ambitious task will be left as a subject of future research. In Sec. III we treat the effects of a symmetry-breaking disorder on the cavity polaritons by applying CPT Green's function technique, because this approach is analytical nonperturbative one which provides exact results. Furthermore, the Keldysh formalism could be applied to the cavity polaritons in nonequilibrium conditions.

II. CAVITY POLARITONS IN THE ABSENCE OF A DISORDER - THE MATSUBARA GREEN'S FUNCTION APPROACH

The system under consideration consists of a single QW grown inside a semiconductor MC is an arrangement of two-plane parallel mirrors with reflectivity close to unity. The two infinite and parallel perfect mirrors are perpendicular to z -axis, separated by a distance L_0 , one mirror is at $z = L_0/2$, and the other at $z = -L_0/2$. In what follows we are interested in the case of a single QW extending over $-L/2 < z < L/2$ made from a direct-gap semiconductor with nondegenerate and isotropic bands when the electron-hole motion along the z -direction is confined between two parallel, infinitely high potential barriers. With the perfect confinement approximation the dispersion laws for electrons and holes are $E_c(\mathbf{k}_c, \lambda) = E_g + \mathbf{k}_c^2/2m_c + \pi^2\lambda^2/2m_cL^2$ and $E_v(\mathbf{k}_v, \xi) = -\mathbf{k}_v^2/2m_v - \pi^2\xi^2/2m_vL^2$, respectively. Here m_c (m_v) is the electron (hole) effective mass, E_g is the energy gap, and $\mathbf{k}_{c,v}$ is a two-dimensional (2D) wave vector. $\lambda, \xi = 1, 2, \dots$ denote the quantum number of the states in the infinitely deep wells. In what follows we use the simplest approximation which takes into account only the first electron and hole confined levels, i.e. $\lambda = \xi = 1$. For each photon wave vector there are two possible polarizations: one with transverse electric field (TE), and second, with transverse magnetic field (TM).¹⁶ In what follows we will take into account only the TE modes which interact with transverse polarized excitons. The longitudinal photon modes mediate the Coulomb interaction between the charges in the QW, but we neglect this effect assuming that the Coulomb interaction between the charges in the QW is affected only by the confinement of the charges. The cavity-mode dispersion is $\Omega_s(\mathbf{q}) = c\sqrt{q^2 + (\pi s/L_0)^2}$, where $s = 1, 2, \dots$, and \mathbf{q} is a 2D vector. In the following, we suppose that the only $s = 1$ cavity modes $\Omega(\mathbf{q})$ interact with the electron system.

In terms of the field theory, the transverse and the longitudinal photon modes are described by boson fields $A_\perp(\rho)$ and $A_\parallel(\rho)$, respectively. They interact with the

electron system, described by fermion fields $\psi^+(y)$ and $\psi(x)$. The total action of the system is

$$S = S_0^{(e)} + S_0^{(\omega)} + S^{(e-\omega)}.$$

The actions for non-interacting electrons and photons are

$$S_0^{(e)} = \bar{\psi}(y)G^{(0)-1}(y, x)\psi(x),$$

and

$$S_0^{(\omega)} = \frac{1}{2}A_{\parallel}(\rho)D_{\parallel}^{(0)-1}(\rho, \rho')A_{\parallel}(\rho') + \frac{1}{2}A_{\perp}(\rho)D_{\perp}^{(0)-1}(\rho, \rho')A_{\perp}(\rho'),$$

respectively. The electron-photon interaction is described by

$$S^{(e-\omega)} = \bar{\psi}(y)\Gamma_{\parallel}^{(0)}(y, x | \rho)\psi(x)A_{\parallel}(\rho) + \bar{\psi}(y)\Gamma_{\perp}^{(0)}(y, x | \rho)\psi(x)A_{\perp}(\rho).$$

The composite variables $y = \{\mathbf{r}, u\}$, $x = \{\mathbf{r}', u'\}$, and $\rho = \{\mathbf{R}, v\}$ are defined as follows: $\mathbf{r}, \mathbf{r}', \mathbf{R}$ are 2D radius vectors, and according to imaginary-time (Matsubara) formalism the variable u, u', v range from 0 to $\hbar\beta = \hbar/(kT)$. We set $\hbar = 1$ and we use the summation-integration convention: that repeated variables are summed up or integrated over. $G^{(0)-1}(y, x)$ is the inverse single-particle Green function for non-interacting electrons in a periodic lattice potential $G^{(0)-1}(y, x) = \sum_{\omega_m} e^{-i\omega_m(u-u')}G^{(0)-1}(\mathbf{r}, z, \mathbf{r}', z'; \omega_m)$. The function $G^{(0)-1}(\mathbf{r}, z, \mathbf{r}', z'; \omega_m)$ is defined as a sum of electron $\sum_{\mathbf{k}_c} \varphi_{c, \mathbf{k}_c}^*(\mathbf{r}, z)\varphi_{c, \mathbf{k}_c}(\mathbf{r}', z')G_{cc}^{(0)-1}(\mathbf{k}_c; \omega_m)$ and hole $\sum_{\mathbf{k}_v} \varphi_{v, \mathbf{k}_v}^*(\mathbf{r}, z)\varphi_{v, \mathbf{k}_v}(\mathbf{r}', z')G_{vv}^{(0)-1}(\mathbf{k}_v; \omega_m)$ parts. Here $G_{cc}^{(0)-1}(\mathbf{k}_c; \omega_m) = i\omega_m - [E_c(\mathbf{k}_c, \lambda=1) - \mu_c]$, and $G_{vv}^{(0)-1}(\mathbf{k}_v; \omega_m) = i\omega_m - [E_v(\mathbf{k}_v, \xi=1) - \mu_v]$. The functions $\varphi_{c, \mathbf{k}_c}(\mathbf{r})$ and $\varphi_{v, \mathbf{k}_v}(\mathbf{r})$ are the wave functions of the first electron and hole confined levels, defined by the solutions of the corresponding Schrodinger equations. The electron and hole chemical potentials are denoted by μ_c and μ_v , respectively, and the symbol \sum_{ω_m} is used to denote $\beta^{-1}\sum_m$. For fermion fields we have $\omega_m = (2\pi/\beta)(m + 1/2)$; $m = 0, \pm 1, \pm 2, \dots$. In addition to the lattice potential, the electrons and holes experience a Coulomb interaction, described by the term $\Gamma_{\parallel}^{(0)}D_{\parallel}^{(0)}\Gamma_{\parallel}^{(0)}$:

$$\Gamma_{\parallel}^{(0)}(y_2, x_1 | \rho)D_{\parallel}^{(0)}(\rho, \rho')\Gamma_{\parallel}^{(0)}(y_3, x_4 | \rho') = \delta(u_1 - u_3)\delta(u_2 - u_4) \sum_{\mathbf{k}_i, \mathbf{k}_j, \mathbf{k}'_i, \mathbf{k}'_j, \mathbf{q}} \sum_{i, j} \varphi_{i, \mathbf{k}_i}(\mathbf{r}_1)\varphi_{j, \mathbf{k}_j}^*(\mathbf{r}_2) \times \varphi_{i, \mathbf{k}'_i}(\mathbf{r}_3)\varphi_{j, \mathbf{k}'_j}^*(\mathbf{r}_4)V_0(\mathbf{q}) \left[\delta_{\mathbf{k}_i, \mathbf{k}'_i + \mathbf{q}} + \delta_{\mathbf{k}_j, \mathbf{k}'_j - \mathbf{q}} \right].$$

Here $i, j = \{c, v\}$, $D_{\parallel}^{(0)}$ is the longitudinal part of the photon propagator (in a gauge, when the scalar potential equals zero) and $\Gamma_{\parallel}^{(0)}$ is the vertex. $V_0(\mathbf{q}) =$

$2\pi e^2 f(L|\mathbf{q}|)/|\mathbf{q}|$ denotes the Fourier transform of the 2D unscreened Coulomb potential. The structure factor $f(x)$ takes into account the first confined QW electron and hole levels:

$$f(x) = \frac{3x^2 + 8\pi^2}{x(x^2 + 4\pi^2)} - \frac{32\pi^4[1 - \exp(-x)]}{x^2(x^2 + 4\pi^2)^2}.$$

The inverse transverse photon propagator is:

$$D_{\perp}^{(0)-1}(\rho, \rho') = D_{\perp}^{(0)-1}(\mathbf{R}, v; \mathbf{R}', v') = \frac{1}{A_0} \sum_{\mathbf{q}} \sum_{\omega_p} e^{i[\mathbf{q} \cdot (\mathbf{R} - \mathbf{R}') - \omega_p(v - v')]} D_{\perp}^{(0)-1}(\mathbf{q}, \omega_p). \quad (4)$$

Here A_0 is the area of the cavity, and $D_{\perp}^{(0)-1}(\mathbf{q}, \omega_p) = 2\pi c^2 / [(\omega_p)^2 - \Omega^2(\mathbf{q})]$. The symbol \sum_{ω_p} is used to denote $\beta^{-1}\sum_p$. For boson fields we have $\omega_p = (2\pi/\beta)p$; $p = 0, \pm 1, \pm 2, \dots$. We assume that the electron-hole-photon system is in thermal equilibrium, which means that the poles of the photon retarded Green's function have to be obtained from the corresponding Matsubara Green's function, by the substitution $\omega_p \rightarrow \omega + \mu + i0^+$. Here $\mu = \mu_c - \mu_v$ is the chemical potential of the system. The vertex $\Gamma_{\perp}^{(0)}(y, x | \rho)$ has the following form:

$$\Gamma_{\perp}^{(0)}(y_2, x_1 | \rho) = \Gamma_{\perp}^{(0)}(\mathbf{r}_2, u_2, \mathbf{r}_1, u_1 | \mathbf{R}, v) = \frac{\delta(u_1 - v)\delta(u_1 - u_2)}{c} \sum_{\mathbf{q}} \sum_{i, \mathbf{k}_i, j, \mathbf{k}_j} e^{i\mathbf{q} \cdot \mathbf{R}} \varphi_{j, \mathbf{k}_j}^*(\mathbf{r}_2)\varphi_{i, \mathbf{k}_i}(\mathbf{r}_1) < j, \mathbf{k}_j | \hat{\mathbf{j}}(\mathbf{q}) \cdot \mathbf{n}(\mathbf{q}) | i, \mathbf{k}_i >, \quad (5)$$

where $\hat{\mathbf{j}}(\mathbf{q})$ denotes the single-particle current operator, and $\mathbf{n}(\mathbf{q}) = \mathbf{e}_q \times \mathbf{e}_z$, where $\mathbf{e}_q = \mathbf{q}/q$ and $\mathbf{e}_z = (0, 0, 1)$. It is well-known that all Green's functions can be obtained by functional differentiation from the generating functional $W[J, M] = \ln Z[J, M]$ of the connected Green's function, where

$$Z[J, M] = \int D\mu[\bar{\psi}, \psi, A_{\perp}, A_{\parallel}] \times \exp[S + J_{\perp}(\rho)A_{\perp}(\rho) + J_{\parallel}(\rho)A_{\parallel}(\rho) - \bar{\psi}(y)M(y, x)\psi(x)]. \quad (6)$$

Here J and M are the sources of the corresponding fields. By means of the functional (6) we introduce the following functions (after the functional differentiation one should set $J=M=0$):

Average photon field:

$$R_{\parallel, (\perp)}(\rho) = -\frac{\delta W}{\delta J_{\parallel, (\perp)}(\rho)}; \quad (7)$$

single-particle Green function:

$$G(x, y) = -\frac{\delta W[J, M]}{\delta M(y, x)}; \quad (8)$$

transverse (longitudinal) photon Green function:

$$D_{\perp(\parallel)}(\rho, \rho') = -\frac{\delta^2 W[J, M]}{\delta J_{\perp(\parallel)}(\rho') \delta J_{\perp(\parallel)}(\rho)} = \frac{\delta R_{\perp, \perp}(\rho)}{\delta J_{\perp, \perp}(\rho')}; \quad (9)$$

two-particle electron-hole Green function:

$$K \begin{pmatrix} x & y' \\ y & x' \end{pmatrix} = -\frac{\delta^2 W[J, M]}{\delta M(y', x') \delta M(y, x)} = \frac{\delta G(x, y)}{\delta M(y', x')}; \quad (10)$$

transverse (longitudinal) electron-photon vertex function:

$$\Gamma_{\perp(\parallel)}(y, x | \rho) = -\frac{\delta G^{-1}(y, x)}{\delta J_{\perp(\parallel)}(\rho')} D_{\perp(\parallel)}^{-1}(\rho', \rho). \quad (11)$$

As a consequence of the fact that the measure is invariant under the translations $\bar{\psi} \rightarrow \bar{\psi} + \delta \bar{\psi}$, $A_{\perp, \parallel} \rightarrow A_{\perp, \parallel} + \delta A_{\perp, \parallel}$ we derive the SD equations:¹⁵

$$\begin{aligned} J_{\parallel(\perp)}(\rho) - D_{\parallel(\perp)}^{(0)-1}(\rho, \rho') R_{\parallel(\perp)}(\rho') \\ + \Gamma_{\parallel(\perp)}^{(0)}(y, x | \rho) G(x, y) = 0, \end{aligned} \quad (12)$$

$$G^{-1}(y, x) - G^{(0)-1}(y, x) + M(y, x) + \Sigma(y, x) = 0. \quad (13)$$

The mass operator Σ has the form:

$$\begin{aligned} \Sigma(y, x) = \Gamma_{\perp}^{(0)}(y, x | \rho) R_{\perp}(\rho) + \Gamma_{\parallel}^{(0)}(y, x | \rho) R_{\parallel}(\rho) \\ - \Gamma_{\parallel}^{(0)}(y, x' | \rho) G(x', y') \Gamma_{\parallel}(y', x | \rho') D_{\parallel}(\rho, \rho') \\ - \Gamma_{\perp}^{(0)}(y, x' | \rho) G(x', y') \Gamma_{\perp}(y', x | \rho') D_{\perp}(\rho, \rho'). \end{aligned} \quad (14)$$

The charge neutrality leads to the photon field $R_{\parallel}(\rho)$ that vanishes identically. Thus, all terms proportional to $R_{\parallel}(\rho)$ (or $D_{\parallel}(\rho, \rho') \Gamma_{\parallel}^{(0)}(y, x | \rho') G(x, y)$) does not need to be taken into account because of the global neutrality of the electron-hole system. In what follows we assume the so-called Hartree-Fock approximation in which the mass-operator has the form

$$\begin{aligned} \Sigma(y, x) = \Gamma_{\perp}^{(0)}(y, x | \rho) R_{\perp}(\rho) - \\ \Gamma_{\parallel}^{(0)}(y, x' | \rho) G(x', y') \Gamma_{\parallel}(y', x | \rho') D_{\parallel}(\rho, \rho'). \end{aligned} \quad (15)$$

The first and the second terms in (15) are called the Hartree term and the Fock term, respectively. The presence of Bose-condensed polaritons modifies the single-particle Greens functions, and therefore one has to consider the so-called normal $G_{cc}^{-1}(\mathbf{k}; \omega_m) = G_{cc}^{(0)-1}(\mathbf{k}; \omega_m) - \Sigma_{cc}(\mathbf{k})$ and $G_{vv}^{-1}(\mathbf{k}; \omega_m) = G_{vv}^{(0)-1}(\mathbf{k}; \omega_m) - \Sigma_{vv}(\mathbf{k})$, and anomalous $G_{cv}^{-1}(\mathbf{k}; \omega_m) = G_{vc}^{-1}(\mathbf{k}; \omega_m) = -\Delta(\mathbf{k})$ single-particle Green's functions (the spin degrees of freedom are not included). The diagonal parts of the mass operator in the Hartree-Fock approximation are as follows: $\Sigma_{cc}(\mathbf{k}) = \sum_{\mathbf{q}} V(\mathbf{k} - \mathbf{q}) \sum_{\omega_m} G_{cc}(\mathbf{q}; \omega_m)$, $\Sigma_{vv}(\mathbf{k}) = \sum_{\mathbf{q}} V(\mathbf{k} - \mathbf{q}) \sum_{\omega_m} G_{vv}(\mathbf{q}; \omega_m)$. Here $V(\mathbf{q}) = 2\pi e^2 f(L|\mathbf{q}|)/\epsilon_{\infty}|\mathbf{q}|$

denotes the screened Coulomb potential. Using (15) we calculate for the non-diagonal parts of the mass operator in the Hartree-Fock approximation:

$$\Delta(\mathbf{k}) = \sum_{\mathbf{q}} \Gamma_{\perp}^{(0)}(\mathbf{q}, \mathbf{k}) R_{\perp}(\mathbf{q}) + \Delta_{exc}(\mathbf{k}), \quad (16)$$

where Δ is the order parameter for the system. The first (Hartree) and the second (Fock) terms in (16) represent the photonic and the excitonic contributions to the order parameter, respectively. The exact form of $\Gamma_{\perp}^{(0)}(\mathbf{q}, \mathbf{k})$ can be calculated by means of (4) and (5), but we assume that the photons are coupled to the electron-hole system through the local interaction, i.e. $\Gamma_{\perp}^{(0)}(\mathbf{q}, \mathbf{k}) = g(\mathbf{q} - \mathbf{k})$. In this approximation the photon field R_{\perp} and the excitonic order Δ_{exc} parameter are defined as follows:

$$R_{\perp}(\mathbf{k}) = \sum_{\mathbf{q}} g(\mathbf{k} - \mathbf{q}) \sum_{\omega_m} G_{cv}(\mathbf{q}; \omega_m) / (\Omega^2(\mathbf{k}) - \mu^2), \quad (17)$$

$$\Delta_{exc}(\mathbf{k}) = \sum_{\mathbf{q}} V(\mathbf{k} - \mathbf{q}) \sum_{\omega_m} G_{cv}(\mathbf{q}; \omega_m). \quad (18)$$

The photonic and the excitonic order parameters are not independent. Using Eq. (18) for the excitonic order parameter, we calculate $\sum_{\omega_m} G_{cv}(\mathbf{q}; \omega_m) = \sum_{\mathbf{k}} V^{-1}(\mathbf{q} - \mathbf{k}) \Delta_{exc}(\mathbf{k})$, and therefore, we obtain the following constraint:

$$R_{\perp}(\mathbf{k}) = \frac{1}{\Omega(\mathbf{k})^2 - \mu^2} \sum_{\mathbf{q}, \mathbf{p}} g(\mathbf{k} - \mathbf{q}) V^{-1}(\mathbf{q} - \mathbf{p}) \Delta_{exc}(\mathbf{p}), \quad (19)$$

where we have introduced a function V^{-1} defined by $\sum_{\mathbf{p}} V(\mathbf{k} - \mathbf{p}) V^{-1}(\mathbf{p} - \mathbf{q}) = \delta(\mathbf{k} - \mathbf{q})$.

In the Hartree-Fock approximation the normal single-particle Green's functions are:¹⁵

$$\begin{aligned} G_{cc}(\mathbf{k}; \omega_m) &= \left[\frac{u_{\mathbf{k}}^2}{i\omega_m - \omega_+(\mathbf{k})} + \frac{v_{\mathbf{k}}^2}{i\omega_m - \omega_-(\mathbf{k})} \right], \\ G_{vv}(\mathbf{k}; \omega_m) &= \left[\frac{v_{\mathbf{k}}^2}{i\omega_m - \omega_+(\mathbf{k})} + \frac{u_{\mathbf{k}}^2}{i\omega_m - \omega_-(\mathbf{k})} \right], \quad (20) \\ G_{cv}(\mathbf{k}; \omega_m) &= G_{vc}(\mathbf{k}; \omega_m) = \\ &= u_{\mathbf{k}} v_{\mathbf{k}} \left[\frac{1}{i\omega_m - \omega_+(\mathbf{k})} - \frac{1}{i\omega_m - \omega_-(\mathbf{k})} \right]. \end{aligned}$$

Here, the following notations have been used:

$$\begin{aligned} u^2(\mathbf{k}) &= \frac{1}{2} \left[1 + \frac{\eta(\mathbf{k})}{\varepsilon(\mathbf{k})} \right], \quad v^2(\mathbf{k}) = \frac{1}{2} \left[1 - \frac{\eta(\mathbf{k})}{\varepsilon(\mathbf{k})} \right], \\ \varepsilon(\mathbf{k}) &= \sqrt{\eta^2(\mathbf{k}) + \Delta^2(\mathbf{k})}, \quad \omega_{\pm}(\mathbf{k}) = \zeta(\mathbf{k}) \pm \varepsilon(\mathbf{k}), \\ \zeta(\mathbf{k}) &= \frac{1}{2} [E_c(\mathbf{k}, 1) + E_v(\mathbf{k}, 1) - \mu_c - \mu_v] \\ &+ \frac{1}{2} \sum_{\mathbf{q}} V(\mathbf{k} - \mathbf{q}) [n_-(\mathbf{q}) - n_+(\mathbf{q})], \\ \eta(\mathbf{k}) &= \frac{1}{2} [E_c(\mathbf{k}, 1) - E_v(\mathbf{k}, 1) - \mu] \\ &- \frac{1}{2} \sum_{\mathbf{q}} V(\mathbf{k} - \mathbf{q}) \left[1 - [1 - n_+(\mathbf{q}) - n_-(\mathbf{q})] \frac{\eta(\mathbf{q})}{\varepsilon(\mathbf{q})} \right], \end{aligned}$$

where $n_{\pm}(\mathbf{k}) = [1 + \exp(\pm\beta\omega_{\pm}(\mathbf{k}))]^{-1}$. Below the critical temperature the order parameter is developed ($\Delta(\mathbf{k}) \neq 0$) and the single-particle excitations are coherent combinations of electron-like $\omega_+(\mathbf{k})$ and hole-like $\omega_-(\mathbf{k})$ excitations, renormalized due to the interaction with the cavity modes. The coefficients $u(\mathbf{k})$ and $v(\mathbf{k})$ which are called coherent factors, give the probability amplitudes of these states in the actual mixture.

It is expected that the BEC phenomenon is not sensitive to the difference in electron and hole effective masses, and therefore, we assume $m_c = m_v = 2m_{exc}$, where $m_{exc}^{-1} = m_c^{-1} + m_v^{-1}$ is the exciton reduced mass. The equal mass assumption simplifies very much the calculations because in this case $\mu_c + \mu_v = E_g$, $\zeta(\mathbf{k}) = 0$, $n_+(\mathbf{k}) = n_-(\mathbf{k}) = [1 + \exp(\beta\varepsilon(\mathbf{k}))]^{-1}$, and $\eta(\mathbf{k})$ is defined by:

$$\eta(\mathbf{k}) = \frac{1}{2} \left[E_g + \frac{\pi^2}{2mL^2} + \frac{\mathbf{k}^2}{2m_{exc}} - \mu \right] - \frac{1}{2} \sum_{\mathbf{q}} V(\mathbf{k} - \mathbf{q}) \left[1 - \tanh\left(\frac{\beta\varepsilon(\mathbf{q})}{2}\right) \frac{\eta(\mathbf{q})}{\varepsilon(\mathbf{q})} \right]. \quad (21)$$

Note, that in our approach $\eta(\mathbf{k})$ is defined self-consistently by the solution of Eq. (21), while MSL neglected the contributions due to the diagonal parts ($\Sigma_{cc}(\mathbf{k})$ and $\Sigma_{vv}(\mathbf{k})$) of the mass operator, using the following expression:

$$\eta(\mathbf{k}) = \mathbf{k}^2/4m_{exc} - \varepsilon_F, \quad (22)$$

where the effective Fermi energy is defined by $\varepsilon_F = p_F^2/4m_{exc} = (\mu - E_g - \pi^2/2m_{exc}L^2)/2$. In the high-density limit, the neglected diagonal parts of the mass operator are responsible only for small renormalization of the single-particle excitations $\omega_{\pm}(\mathbf{k})$. But, in the low-density limit the diagonal parts of the mass operator are crucial, and therefore, they cannot be neglected.

The order parameter (16) includes both excitonic and photonic contributions:

$$\Delta(\mathbf{k}) = \sum_{\mathbf{q}} g(\mathbf{k} - \mathbf{q}) R_{\perp}(\mathbf{q}) + \Delta_{exc}(\mathbf{k}),$$

and is determined by the constraint (19) and the following BCS self-consistent equation for the excitonic order parameter:

$$\Delta_{exc}(\mathbf{k}) = \sum_{\mathbf{q}} V(\mathbf{k} - \mathbf{q}) \sum_{\omega_m} \frac{\Delta(\mathbf{q})}{\omega_m^2 + \varepsilon^2(\mathbf{q})}. \quad (23)$$

To make contact with the equations of MSL in the absence of a disorder, we assume a high-density limit. In the high-density regime the screened Coulomb interaction could be replaced by a short-range contact interaction with a coupling strength g_c given by angular average over the Fermi surface. Assuming that: (i) $g(\mathbf{q}) = g$, and (ii) the order parameters are space independent

($R_{\perp}(\mathbf{k}) = R\delta_{\mathbf{k},0}$, and $\Delta_{exc}(\mathbf{k}) = \Delta_{exc}\delta_{\mathbf{k},0}$), we obtain the total order parameter $\Delta = gR + \Delta_{exc}$, where

$$\Delta_{exc} = g_c \sum_{\mathbf{k}} \sum_{\omega_m} \frac{\Delta}{\omega_m^2 + \varepsilon^2(\mathbf{k})}.$$

While the chemical potential μ does not exceed the cavity-mode energy ω_c at $\mathbf{k} = 0$, the constrain (19) assumes the form:

$$R = \frac{g}{g_c(\omega_c^2 - \mu^2)} \Delta_{exc}. \quad (24)$$

Similar relationship has been found by MSL, but with $\omega_c - \mu$ in place of $\omega_c^2 - \mu^2$. This is because of the different photon Green's functions, used by MSL. In the high-density regime $\eta(\mathbf{k}) = \mathbf{k}^2/m_{exc} - \varepsilon_F$, and Eq. (23) assumes the form of the BCS gap equation for a superconductor:

$$1 = \left(g_c + \frac{g^2}{\omega_c^2 - \mu^2} \right) \sum_{\mathbf{k}} \frac{1}{2\varepsilon(\mathbf{k})} \tanh\left(\frac{\beta\varepsilon(\mathbf{k})}{2}\right). \quad (25)$$

When $g = 0$, the last equation assumes the form (1) with $\alpha = 0$.

The chemical potential μ is a nontrivial function of the total number of excitations $N = N_{ph} + N_{exc}$ in the condensate and should be calculated by solving the BCS equation self-consistently. The total number of photons is $N_{ph} = \sum_{\mathbf{k}} R_{\perp}^2(\mathbf{k})$. The number of condensed electron-hole pairs N_{exc} is:

$$N_{exc} = \sum_{\mathbf{k}} \left[1 - \tanh\left(\frac{\beta\varepsilon(\mathbf{k})}{2}\right) \frac{\eta(\mathbf{k})}{\varepsilon(\mathbf{k})} \right]. \quad (26)$$

Our next step is to show that the photon Green's function and the two-particle electron-hole Green's function have common poles - the excitation spectrum in the presence of a condensed phase. To prove the last we introduce a Legendre transform

$$V[R, G] = W[J, M] + J_{\alpha}(\rho) R_{\alpha}(\rho) + M(y, x) G(x, y). \quad (27)$$

The repeated Greek index α denotes summation over the parallel \parallel and the perpendicular \perp components of the corresponding quantity. By means of the above Legendre transform, we derive the following exact equations:¹⁵

$$\begin{aligned} D_{\perp}(\rho, \rho') &= D_{\perp}^{(0)}((\rho, \rho')) + D_{\perp}^{(0)}(\rho, \rho'') \Gamma_{\perp}^{(0)}((y, x | \rho'')) \\ K \begin{pmatrix} x & y' \\ y & x' \end{pmatrix} \Gamma_{\perp}^{(0)}(y', x' | \rho''') D_{\perp}^{(0)}(\rho''', \rho'), \end{aligned} \quad (28)$$

$$\begin{aligned} \frac{\delta G(x, y)}{\delta J_{\perp}(\rho)} &= \\ K^{(0)} \begin{pmatrix} x & y' \\ y & x' \end{pmatrix} \Gamma_{\perp}(y', x' | \rho') D_{\perp}(\rho', \rho) &= \\ K \begin{pmatrix} x & y' \\ y & x' \end{pmatrix} \Gamma_{\perp}^{(0)}(y', x' | \rho') D_{\perp}^{(0)}(\rho', \rho), \end{aligned} \quad (29)$$

$$K^{-1} \begin{pmatrix} y & x' \\ x & y' \end{pmatrix} = K^{(0)-1} \begin{pmatrix} y & x' \\ x & y' \end{pmatrix} - I \begin{pmatrix} y & x' \\ x & y' \end{pmatrix}, \quad (30)$$

where $K^{(0)} \begin{pmatrix} x & y' \\ y & x' \end{pmatrix} = G(x, y')G(x', y)$ is the free two-particle propagator and the kernel of the BS equation (30) is given by:

$$I \begin{pmatrix} y & x' \\ x & y' \end{pmatrix} = \frac{\delta \Sigma(y, x)}{\delta M(y', x')} + \Gamma_{\alpha}^{(0)}(y, x | \rho) D_{\alpha}^{(0)}(\rho, \rho') \Gamma_{\alpha}^{(0)}(y', x' | \rho'). \quad (31)$$

There are two important conclusions that could be extracted from the above equations. The first one follows from Eq. (29). This equation clearly indicates that the Fock term in Eq. (15) is related to the two-particle Green's function, so we can write the mass operator in the following form:

$$\begin{aligned} \Sigma(y, x) &= \Gamma_{\alpha}^{(0)}(y, x | \rho) R_{\alpha}(\rho) \\ &- \Gamma_{\alpha}^{(0)}(y, x' | \rho) K \begin{pmatrix} x' & y'' \\ y' & x'' \end{pmatrix} \\ &\Gamma_{\alpha}^{(0)}(y'', x'' | \rho') D_{\alpha}^{(0)}(\rho, \rho') G^{-1}(y', x). \end{aligned} \quad (32)$$

The second conclusion follows from Eq. (28). Obviously, the transverse photon Green's function and the two-particle Green's function have common poles - cavity polaritons. The poles of the transverse photon Green's function are defined by the solutions of the Maxwell equations for a transverse wave $\epsilon(\mathbf{Q}, \omega) = \Omega^2(\mathbf{Q})/\omega^2$, where the dielectric function $\epsilon(\mathbf{Q}, \omega)$ in the case of a single excitonic resonance at energy $E_l(\mathbf{Q})$ is:

$$\epsilon(\mathbf{Q}, \omega) = \epsilon_b + \frac{d}{E_l^2(\mathbf{Q}) - \omega^2 - 2i\omega\gamma_0}.$$

Here ϵ_b is the background dielectric constant, γ_0 is the broadening of the excitonic resonance, and d is proportional to the corresponding oscillator strength. In principle, the excitonic resonance energy and the oscillator strength can be calculated by solving a set of coupled BS equations for the energy and the wavefunctions of the quantum-well excitons. These equations are similar to equations (42) and (43) from our earlier paper.¹⁵ The only change that should be done is related to the existence of an extra term, $\Gamma_{\perp}^{(0)} D_{\perp}^{(0)} \Gamma_{\perp}^{(0)} G$, in the mass operator. The last term contributes to the exchange interaction in the similar manner as the term $\Gamma_{\parallel}^{(0)} D_{\parallel}^{(0)} \Gamma_{\parallel}^{(0)} G$ generates the analytical exchange interaction between electrons and holes. The exchange interactions are important only for the fine structure of exciton levels, and therefore, we neglect the exchange interaction terms. As a result, we obtain the BS equations similar to the case of the excitonic condensate. Generally speaking, to calculate the excitation spectrum in the presence of a condensed phase, one has to solve the BCS and the BS equations simultaneously, taking into account the fact that the chemical potential depends on the number of excitons and photons

in the condensed phase. Such an ambitious task will be left as a subject of future research.

We finish this section with a brief discussion of the so-called Thouless criterion.¹⁷ In the case of superconductivity this criterion says that below the critical temperature T_c the T-matrix has a pole at zero frequency and zero momentum (the existence of Goldstone mode below T_c). To check whether the Thouless criterion works in the case of condensed cavity polaritons we follow the method based on the Ward identities.¹⁸ Taking into account the fact that in the case of MC polaritons the mass operator depends on the photon field $R_{\perp(\parallel)}$ and the Green's function G , we first invert the SD equations to express the sources $J_{\alpha}(\rho)$ and $M(y, x)$ as functionals of the field R_{α} and the Green's function G . Second, we assume that there exists a continuous transformation, for example, a rotation in order-parameter space, which depends continuously on the parameter λ . If the system is invariant under this transformation, then the variation of the Legendre transform implied by the transformation is equal to zero, i.e. $\delta_{\lambda} V = 0$. The BS equation for the two-particle Green's function $K = K^{(0)} + K^{(0)} I K$ can be rewritten in terms of the many-particle T-matrix, $T = I + I K^{(0)} T$, in the form $K = K^{(0)} + K^{(0)} T K^{(0)}$. By means of the last form of the BS equation and the definition (10) we calculate the variation of the inverse Green's function $\delta_{\lambda} G^{-1} = -(1 + T K^{(0)}) \delta_{\lambda} M$. Using the SD equations we find $\delta_{\lambda} G^{-1} = -\delta_{\lambda} M - \delta_{\lambda} \Sigma$, and therefore, $T^{-1} \delta_{\lambda} \Sigma = K^{(0)} \delta_{\lambda} M$. But, according to (27) we calculate $\delta_{\lambda} M = \frac{\delta}{\delta G} (\delta_{\lambda} V) = 0$, and therefore, $T^{-1} \delta_{\lambda} \Sigma = 0$. Above the critical temperature T_c the order parameter is zero, and hence, $\delta_{\lambda} \Sigma = 0$. Thus, $T^{-1} \delta_{\lambda} \Sigma = 0$ is satisfied trivially. Below T_c the order parameter is nonzero and $\delta_{\lambda} \Sigma \neq 0$, which requires that the inverse T-matrix has a zero eigenvalue. Thus, we conclude that below the critical temperature T_c , the T-matrix must have a pole at zero energy and zero momentum. The existence of Goldstone mode below T_c indicates that the formation of a condensate in MC is possible.

III. CAVITY POLARITONS IN THE PRESENCE OF A SYMMETRY-BREAKING DISORDER - THE KELDYSH GREEN'S FUNCTION APPROACH

In the presence of a disorder we define the Green's functions as an average of the time-ordered products of the fields, but the average includes both the quantum and the disorder averaging. We use $\langle F \rangle$ for the quantum averaging, and \overline{F} for the disorder averaging. The Matsubara Green's function approach discussed in the previous Section, cannot be directly applied to the cavity polaritons in the presence of a disorder, because one has to calculate $\langle \ln F \rangle$. Nevertheless, for a given disorder configuration we can write the relationship between the random photon Green's function G and the random two-particle Green's function K , similar to Eq. (28). After

that, the disorder averaging replaces the random Green's functions on the both sides of Eq. (28) by their averages. Thus, we obtain that average photon \overline{D} and average two-particle Green's function \overline{K} have common poles - the cavity polaritons in the presence of a disorder. The next question to be answered is about the possibility to observe a condensate in MC in the presence of a disorder. To answer this question we have to examine the non-diagonal parts of the average single-particle Green's function. The last can be obtained by solving the SD equations. We shall use the Keldysh technique to derive the SD equations in the presence of a disorder, because the CPT approach allows us to perform the average over the random potential exactly, i.e. non-perturbatively. Within this approach we have boson (photon) longitudinal and transverse fields $A_\alpha(\underline{z}) = A_{\parallel,\perp}(\mathbf{R}, \underline{t}'')$ interacting with a fermion (electron) field $\psi^+(\underline{y}) = \psi_j^+(\mathbf{r}, \underline{t})$, or $\psi(\underline{x}) = \psi_{j'}(\mathbf{r}', \underline{t}')$, in the presence of disorder. The variables $\underline{y}, \underline{x}$ and \underline{z} are composite variables $\underline{y} = \{\mathbf{r}, \underline{t}, j\}$, $\underline{x} = \{\mathbf{r}', \underline{t}', j'\}$, $\underline{z} = \{\mathbf{R}, \underline{t}''\}$, where $\mathbf{r}, \mathbf{r}', \mathbf{R}$ are the corresponding 2D radius vectors. The index $j = 1$ (or $j = c$) denotes the electron states, and $j = 2$ (or $j = v$) denotes the hole states. For simplicity, we suppose that the electrons are spinless. At a zero temperature the total action of the system is $S = S^{(e)} + S_0^{(\omega)} + S^{(e-\omega)}$. To incorporate the effect of disorder we introduce random static symmetry-breaking (charge-dependent) potential $V(\mathbf{r})$. In the presence of a disorder the action which corresponds to the electron system assumes the form:

$$S^{(e)} = \psi^+(\underline{y})[G^{(0)-1}(\underline{y}, \underline{x}) - V(\underline{y}, \underline{x})]\psi(\underline{x}),$$

where $V(\underline{y}, \underline{x}) = V(\mathbf{r}, \underline{t}, j; \mathbf{r}', \underline{t}', j') = V(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}')\delta(\underline{t} - \underline{t}')\delta_{jj'}$. The actual potential $V(\mathbf{r})$ is unknown, but we assume that it obeys Gaussian statistics such that:

$$\overline{V(\mathbf{r})} = 0, \quad \overline{V(\mathbf{r})V(\mathbf{r}')} = \Lambda\delta(\mathbf{r} - \mathbf{r}'), \quad (33)$$

where $\Lambda = (2\pi\nu\tau)^{-1}$, $\nu = m_{exc}/\pi$ is the 2D density of states, and τ is the corresponding scattering time. The disorder averaging in (33) is defined for any functional $F[V]$ by the functional integral:

$$\overline{F} = \int DV F[V] \exp[-\frac{1}{2\Lambda} \int V^2(\mathbf{r})d\mathbf{r}]. \quad (34)$$

The actions $S_0^{(\omega)}$ and $S^{(e-\omega)}$ are given by:

$$S_0^{(\omega)} = \frac{1}{2}A_\alpha(\underline{z})D_\alpha^{(0)-1}(\underline{z}, \underline{z}')A_\alpha(\underline{z}'),$$

$$S^{(e-\omega)} = \psi^+(\underline{y})\Gamma_\alpha^{(0)}(\underline{y}, \underline{x} | \underline{z})\psi(\underline{x})A_\alpha(\underline{z}).$$

In the CPT formalism, the Green's functions are defined by means of two time orderings in the same formula. In what follows we use the single-time representation. In this representation the two time orderings are replaced by a single time ordering along the Keldysh contour which at a zero temperature consists of two branches: the right-going (+) from $-\infty$ to ∞ and the left-going (-) from ∞ to $-\infty$. The symbol \underline{t} means that the time integral $\int d\underline{t}$ along the Keldysh contour could be written as two usual integrals, i.e. $\int d\underline{t} = \int_{-\infty}^{\infty} dt^+ - \int_{-\infty}^{\infty} dt^-$. In other words, the time variable \underline{t} on the positive branch equals $\underline{t} = t^+$, and $\underline{t} = t^-$ on the negative branch. It should be mentioned that our equations are valid for both nonequilibrium and equilibrium conditions, but we do not discuss time-dependent phenomena on an ultra-fast scale. Instead, we intend to describe steady-state phenomenon, such as the light propagation in crystal in terms of excitonic polaritons. In the steady-state regime all quantities depend on the relative time $\underline{t}' - \underline{t}''$. The inverse free propagator $G^{(0)-1}(\underline{y}, \underline{x}) = G_{jj'}^{(0)-1}(\mathbf{r}, \underline{t}; \mathbf{r}', \underline{t}')$ is defined as follows:

$$\begin{aligned} G_{jj'}^{(0)-1}(\mathbf{r}, \underline{t}; \mathbf{r}', \underline{t}') &= \delta(\underline{t} - \underline{t}')\delta_{jj'}\delta_{jc} \sum_{\mathbf{k}_c} \varphi_{c,\mathbf{k}_c}(\mathbf{r})\varphi_{c,\mathbf{k}_c}^*(\mathbf{r}') \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} G_c^{(0)-1}(\mathbf{k}_c; \omega) e^{i\omega\underline{t}} \\ &+ \delta(\underline{t} - \underline{t}')\delta_{jj'}\delta_{jv} \sum_{\mathbf{k}_v} \varphi_{v,\mathbf{k}_v}(\mathbf{r})\varphi_{v,\mathbf{k}_v}^*(\mathbf{r}') \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} G_v^{(0)-1}(\mathbf{k}_v; \omega) e^{i\omega\underline{t}} \end{aligned} \quad (35)$$

Here $G_c^{(0)-1}(\mathbf{k}_c; \omega) = \omega - [E_c(\mathbf{k}_c, \lambda = 1) - \mu_c] + i0^+$, $G_v^{(0)-1}(\mathbf{k}_v; \omega) = \omega - [E_v(\mathbf{k}_v, \xi = 1) - \mu_v] - i0^+$ are the inverse zero-temperature free electron and hole propagators.

In addition to the lattice and the random potentials, the electrons and holes experience a Coulomb interaction, described by the term $\Gamma_{\parallel}^{(0)} D_{\parallel}^{(0)} \Gamma_{\parallel}^{(0)}$:

$$\begin{aligned} \Gamma_{\parallel}^{(0)}(\underline{y}_2, \underline{x}_1 | \underline{z}) D_{\parallel}^{(0)}(\underline{z}, \underline{z}') \Gamma_{\parallel}^{(0)}(\underline{y}_3, \underline{x}_4 | \underline{z}') &= \delta(\underline{t}_1 - \underline{t}_3)\delta(\underline{t}_2 - \underline{t}_4) \sum_{j,j'} \sum_{\mathbf{k}_j, \mathbf{p}_{j'}, \mathbf{q}} V_0(\mathbf{q}) \varphi_{j,\mathbf{k}_j}(\mathbf{r}_1) \varphi_{j,\mathbf{k}_j+\mathbf{q}}^*(\mathbf{r}_2) \times \\ &\varphi_{j',\mathbf{p}_{j'}}^*(\mathbf{r}_3) \varphi_{j',\mathbf{p}_{j'}-\mathbf{q}}(\mathbf{r}_4). \end{aligned} \quad (36)$$

Here $D_{\parallel}^{(0)}$ is the longitudinal part of the photon propagator (in a gauge, when the scalar potential equals zero) and $\Gamma_{\parallel}^{(0)}$ is the vertex. $V_0(\mathbf{q})$ denotes the Fourier transform of the 2D bare Coulomb potential, and has been defined in

Sec. II.

The inverse transverse photon propagator is:

$$D_{\perp}^{(0)-1}(\underline{z}, \underline{z}') = D_{\perp}^{(0)-1}(\mathbf{R}, \underline{t}; \mathbf{R}', \underline{t}') = \frac{\delta(\underline{t} - \underline{t}')}{A} \sum_{\mathbf{q}} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{i[\mathbf{q} \cdot (\mathbf{R} - \mathbf{R}') - \omega \underline{t}]} D_{\perp}^{(0)-1}(\mathbf{q}; \omega), \quad (37)$$

Here A is the area of the cavity, and $D_{\perp}^{(0)-1}(\mathbf{q}, \omega) = 2\pi c^2 / [(\omega - \mu)^2 - \Omega^2(\mathbf{q}) + i0^+]$. The vertex $\Gamma_{\perp}^{(0)}(\underline{y}, \underline{x} | \underline{z})$ has the following form:

$$\Gamma_{\perp}^{(0)}(\underline{y}_2, \underline{x}_1 | \underline{z}) = \Gamma_{\perp jj'}^{(0)}(\mathbf{r}_2, \underline{t}_2, \mathbf{r}_1, \underline{t}_1 | \mathbf{R}, \underline{t}) = \frac{\delta(\underline{t}_1 - \underline{t})\delta(\underline{t}_1 - \underline{t}_2)}{c} \sum_{\mathbf{q}} \sum_{\mathbf{k}_j, \mathbf{p}_{j'}} e^{i\mathbf{q} \cdot \mathbf{R}} \varphi_{j, \mathbf{k}_j}^*(\mathbf{r}_2) \varphi_{j', \mathbf{p}_{j'}}(\mathbf{r}_1) < j, \mathbf{k}_j | \hat{\mathbf{j}}(\mathbf{q}) \cdot \mathbf{n}(\mathbf{q}) | j', \mathbf{p}_{j'} >, \quad (38)$$

where $\hat{\mathbf{j}}(\mathbf{q})$ denotes the single-particle current operator, and $\mathbf{n}(\mathbf{q}) = \mathbf{e}_{\mathbf{q}} \times \mathbf{e}_z$, where $\mathbf{e}_{\mathbf{q}} = \mathbf{q}/q$ and $\mathbf{e}_z = (0, 0, 1)$. Let us introduce the generating functional $\mathbf{W}[J, M; V]$ of the connected Green functions:

$$\mathbf{W}[J, M; V] = -i \ln \mathbf{Z}[J, M; V], \quad (39)$$

where $J = J_{\parallel, \perp}(\underline{z})$ and $M = M(\underline{y}, \underline{x})$ are the sources, and the functional $\mathbf{Z}[J, M; V]$ has the form:

$$\mathbf{Z}[J, M; V] = \int D\mu \exp\{i[S + J_{\parallel}(\underline{z})A_{\parallel}(\underline{z}) + J_{\perp}(\underline{z})A_{\perp}(\underline{z}) - \psi^+(\underline{y})M(\underline{y}, \underline{x})\psi(\underline{x})]\}. \quad (40)$$

Here $D\mu = CD\psi^+D\psi DA$ denotes the functional measure. The success of the Keldysh technique is based on the fact that the normalization constant C is disorder-independent. Thus, we assume that C is chosen in the manner that $\mathbf{Z}[J = 0, M = 0; V] = 1$. It is clear that because J and M do not have the same behavior on the forward and backward parts of the Keldysh contour, the generating functional is not equal to unity if the sources are not nullified.

By means of the generating functional of the connected Green functions we introduce the following average quantities:

Photon field $R_{\alpha}(\underline{z})$ (in what follows $\alpha = \parallel$, or $\alpha = \perp$):

$$R_{\alpha}(\underline{z}) = \frac{\delta \mathbf{W}[J, M; V]}{\delta J_{\alpha}(\underline{z})} \Big|_{J=M=0}; \quad (41)$$

photon Green's function $D_{\alpha}(\underline{z}, \underline{z}')$:

$$D_{\perp}(\parallel)(\underline{z}, \underline{z}') = - \frac{\delta^2 \mathbf{W}[J, M; V]}{\delta J_{\perp}(\parallel)(\underline{z}) \delta J_{\perp}(\parallel)(\underline{z}')} \Big|_{J=M=0}; \quad (42)$$

single-particle Green's function $G(\underline{x}, \underline{y})$:

$$G(\underline{x}, \underline{y}) = -i \frac{\delta \mathbf{W}[J, M, V]}{\delta M(\underline{y}, \underline{x})} \Big|_{J=M=0}; \quad (43)$$

two-particle electron-hole Green's function $K \left(\begin{smallmatrix} \underline{x} & \underline{y}' \\ \underline{y} & \underline{x}' \end{smallmatrix} \right)$:

$$K \left(\begin{smallmatrix} \underline{x} & \underline{y}' \\ \underline{y} & \underline{x}' \end{smallmatrix} \right) = - \frac{\delta^2 \mathbf{W}[J, M; V]}{\delta M(\underline{y}, \underline{x}) \delta M(\underline{y}', \underline{x}')} \Big|_{J=M=0}. \quad (44)$$

Evidently, we have four single-electron Green's functions $G^{\eta_1 \eta_2}$ and four photon Green's functions $D^{\eta_1 \eta_2}$, where $\eta_1, \eta_2 = +$ or $-$ depending on whether the time variable \underline{t} is on the positive branch or on the negative branch. Note, that the time integration in expressions like $C(\underline{x}_1, \underline{x}_3) = A(\underline{x}_1, \underline{x}_2)B(\underline{x}_2, \underline{x}_3)$ follow the convention:

$$\begin{aligned} C^{\eta_1 \eta_2}(\underline{x}_1, \underline{x}_3) &= C(\mathbf{r}_1, t_1^{\eta_1}; \mathbf{r}_3, t_3^{\eta_2}) \\ &= \int d\mathbf{r}_2 \int_{-\infty}^{\infty} dt_2^+ A(\mathbf{r}_1, t_1^{\eta_1}; \mathbf{r}_2, t_2^+) B(\mathbf{r}_2, t_2^+; \mathbf{r}_3, t_3^{\eta_2}) \\ &\quad - \int d\mathbf{r}_2 \int_{-\infty}^{\infty} dt_2^- A(\mathbf{r}_1, t_1^{\eta_1}; \mathbf{r}_2, t_2^-) B(\mathbf{r}_2, t_2^-; \mathbf{r}_3, t_3^{\eta_2}). \end{aligned} \quad (45)$$

Due to the time-translational invariance $K \left(\begin{smallmatrix} \mathbf{r}_1, \underline{t}_1 & \mathbf{r}_3, \underline{t}_3 \\ \mathbf{r}_2, \underline{t}_2 & \mathbf{r}_4, \underline{t}_4 \end{smallmatrix} \right)$ depends on $\underline{t}_{12} = \underline{t}_1 - \underline{t}_2$, $\underline{t}_{43} = \underline{t}_4 - \underline{t}_3$ and $\underline{t}_{31} = \underline{t}_3 - \underline{t}_1$. In what follows, we shall see that our equations will involve the two-particle Green's functions with $\underline{t}_{12} = \underline{t}_{43} = 0$, and therefore, we have four different two-particle Green's functions $K^{\eta_1 \eta_2} = K \left(\begin{smallmatrix} \mathbf{r}_1, \underline{t}_1^{\eta_1} & \mathbf{r}_3, \underline{t}_3^{\eta_2} \\ \mathbf{r}_2, \underline{t}_1^{\eta_1} & \mathbf{r}_4, \underline{t}_3^{\eta_2} \end{smallmatrix} \right)$. The corresponding retarded Green's functions G^R, D^R, K^R , for example, G^R , can be expressed as $G^R = G^{--} - G^{-+} = G^{+-} - G^{++}$. The Keldysh technique allows us to perform the disorder averaging. The resulting equations for the average photon field $R_{\alpha}(\underline{z})$ and the average single-particle Green's function $G(\underline{x}, \underline{y})$ are as follows:

$$R_{\alpha}(\underline{z}) = -i \frac{\delta Z[J, M]}{\delta J_{\alpha}(\underline{z})} \Big|_{J=M=0}, \quad (46)$$

$$G(\underline{x}, \underline{y}) = - \frac{\delta Z[J, M]}{\delta M(\underline{y}, \underline{x})} \Big|_{J=M=0}. \quad (47)$$

where the average generating functional $Z[J, M] = \overline{\mathbf{Z}[J, M; V]}$ is defined by the equation:

$$Z[J, M] = \int D\mu \exp\{\imath[\psi^+(\underline{y})G^{(0)-1}(\underline{y}, \underline{x})\psi(\underline{x}) + \frac{1}{2}A_\alpha(\underline{z})D_\alpha^{(0)-1}(\underline{z}, \underline{z}')A_\alpha(\underline{z}') + \psi^+(\underline{y})\Gamma_\alpha^{(0)}(\underline{y}, \underline{x} | \underline{z})\psi(\underline{x})A_\alpha(\underline{z}) + \frac{\imath}{2}\Lambda\psi_j^+(\mathbf{r}, \underline{t})\psi_j(\mathbf{r}, \underline{t})\psi_{j'}^+(\mathbf{r}, \underline{t}')\psi_{j'}(\mathbf{r}, \underline{t}') + J_\alpha(\underline{z})A_\alpha(\underline{z}) - \psi^+(\underline{y})M(\underline{y}, \underline{x})\psi(\underline{x})]\}. \quad (48)$$

To calculate R and G one has to know the functional $Z[J, M]$. Note that $Z[J = 0, M = 0] = \overline{\mathbf{Z}[J = 0, M = 0; V]} = 1$. Let us define the generating functional $W[J, M] = -\imath \ln Z[J, M]$. By means of this definition we introduce two new functionals: $R_\alpha(\underline{z}; J, M) = \delta W[J, M] / \delta J_\alpha(\underline{z})$ and $G(\underline{x}, \underline{y}; J, M) = -\imath \delta W[J, M] / \delta M(\underline{y}, \underline{x})$. When the sources are nullified, the new functionals are equal to the average photon field $R_\alpha(\underline{z})$ and to the average Green's function $G(\underline{x}, \underline{y})$, respectively:

$$R_\alpha(\underline{z}; J, M)|_{J=M=0} = \frac{\delta W[J, M]}{\delta J_\alpha(\underline{z})}|_{J=M=0} = -\imath \frac{1}{Z[J, M]}|_{J=M=0} \frac{\delta Z[J, M]}{\delta J_\alpha(\underline{z})}|_{J=M=0} = -\imath \frac{\delta Z[J, M]}{\delta J_\alpha(\underline{z})}|_{J=M=0} = R_\alpha(\underline{z})$$

$$G(\underline{x}, \underline{y}; J, M)|_{J=M=0} = -\imath \frac{\delta W[J, M]}{\delta M(\underline{y}, \underline{x})}|_{J=M=0} = -\frac{1}{Z[J, M]}|_{J=M=0} \frac{\delta Z[J, M]}{\delta M(\underline{y}, \underline{x})}|_{J=M=0} = -\frac{\delta Z[J, M]}{\delta M(\underline{y}, \underline{x})}|_{J=M=0} = G(\underline{x}, \underline{y})$$

The next step is to derive the SD equations for the corresponding average quantities using the fact that the functional measure in (48) is invariant under the translations $\psi^+ \rightarrow \psi^+ + \delta\psi^+$, $A \rightarrow A + \delta A$. This assumption yields highly nontrivial relations among generating functionals and their derivatives which are as follows:

$$0 = J_{\parallel(\perp)}(\underline{z}) + D_{\parallel(\perp)}^{(0)-1}(\underline{z}, \underline{z}')R_{\parallel(\perp)}(\underline{z}'; J, M) - \imath \Gamma_{\parallel(\perp)}^{(0)}(\underline{y}, \underline{x} | \underline{z})G(\underline{x}, \underline{y}; J, M), \quad (49)$$

$$G^{-1}(\underline{y}, \underline{x}; J, M) = G^{(0)-1}(\underline{y}, \underline{x}) - M(\underline{y}, \underline{x}) - \Sigma(\underline{y}, \underline{x}; J, M). \quad (50)$$

Here, $\Sigma(\underline{y}, \underline{x}; J, M)$ is a functional of the sources, but when the sources are nullified we obtain the mass operator $\Sigma(\underline{y}, \underline{x})$ for the average single-particle Green's function:

$$\begin{aligned} \Sigma(\underline{y}, \underline{x}) = & -\Gamma_{\parallel}^{(0)}(\underline{y}, \underline{x}' | \underline{z})R_{\parallel}(\underline{z}) - \Gamma_{\perp}^{(0)}(\underline{y}, \underline{x}' | \underline{z})R_{\perp}(\underline{z}) - \Gamma_{\parallel}^{(0)}(\underline{y}, \underline{x}' | \underline{z}) \left(-\imath \frac{\delta G(\underline{x}', \underline{y}'; J, M)}{\delta J_{\parallel}(\underline{z})}|_{J=M=0} \right) G^{-1}(\underline{y}', \underline{x}) \\ & - \Gamma_{\perp}^{(0)}(\underline{y}, \underline{x}' | \underline{z}) \left(-\imath \frac{\delta G(\underline{x}', \underline{y}'; J, M)}{\delta J_{\perp}(\underline{z})}|_{J=M=0} \right) G^{-1}(\underline{y}', \underline{x}) - \imath \Lambda K \begin{pmatrix} \mathbf{r}, \underline{t}, j & \mathbf{r}, \underline{t}^I, j^I \\ \mathbf{r}^{II}, \underline{t}^{II}, j^{II} & \mathbf{r}, \underline{t}^I, j^I \end{pmatrix} G_{j^{II}j'}^{-1}(\mathbf{r}^{II}, \underline{t}^{II}, \mathbf{r}', \underline{t}'). \end{aligned} \quad (51)$$

The term proportional to $R_{\parallel}(\underline{z})$ does not need to be taken into account because of the charge neutrality of the system. The next step is to find the relationship between the mass operator and the average two-particle Green's function. By solving the SD equations (49) and (50), one can obtain the sources J_α and M as functionals of R_α and G . By means of the identity:

$$0 = \frac{\delta J_\alpha(\underline{z})}{\delta M(\underline{y}, \underline{x})} = \frac{\delta J_\alpha(\underline{z})}{\delta G(\underline{x}', \underline{y}')} \frac{\delta G(\underline{x}', \underline{y}')}{\delta M(\underline{y}, \underline{x})} + \frac{\delta J_\alpha(\underline{z})}{\delta R_\beta(\underline{z})} \frac{\delta R_\beta(\underline{z})}{\delta M(\underline{y}, \underline{x})},$$

we calculate

$$-\imath \frac{\delta G(\underline{x}', \underline{y}'; J, M)}{\delta J_{\parallel(\perp)}(\underline{z})}|_{J=M=0} = -\imath \frac{\delta G(\underline{x}', \underline{y}'; J, M)}{\delta M(\underline{y}', \underline{x}')}|_{J=M=0} \Gamma_{\parallel(\perp)}^{(0)}(\underline{y}', \underline{x}' | \underline{z}') D_{\parallel(\perp)}^{(0)}(\underline{z}', \underline{z}). \quad (52)$$

In the absence of a symmetry-breaking disorder the $-\imath \delta G / \delta M$ is nothing but the two-particle Green's function K . However, in the presence of a symmetry-breaking disorder the average generating functional $Z[J, M]$ is not enough to obtain the average photon Green's function and the average two-particle Green's function, because they both can be written as functional derivatives of $Z[J, M]$ plus terms not directly related to the functional $Z[J, M]$ or its derivatives:

$$\begin{aligned} K \begin{pmatrix} \underline{x} & \underline{y}' \\ \underline{y} & \underline{x}' \end{pmatrix} = & \imath \frac{\delta^2 Z[J, M]}{\delta M(\underline{y}, \underline{x}) \delta M(\underline{y}', \underline{x}')}|_{J=M=0} - \overline{\imath G(\underline{x}, \underline{y}; V, J, M) G(\underline{x}', \underline{y}'; V, J, M)}|_{J=M=0} \\ = & -\imath \frac{\delta G(\underline{x}, \underline{y}; J, M)}{\delta M(\underline{y}', \underline{x}')}|_{J=M=0} + \imath G(\underline{x}, \underline{y}) G(\underline{x}', \underline{y}') - \overline{\imath G(\underline{x}, \underline{y}; V, J, M) G(\underline{x}', \underline{y}'; V, J, M)}|_{J=M=0}, \end{aligned} \quad (53)$$

$$\begin{aligned}
D_{\perp(\parallel)}(\underline{z}, \underline{z}') &= i \frac{\delta^2 Z[J, M]}{\delta J_{\perp(\parallel)}(\underline{z}) \delta J_{\perp(\parallel)}(\underline{z}')} \Big|_{J=M=0} + \overline{i R_{\perp(\parallel)}(\underline{z}; V, J, M) R_{\perp(\parallel)}(\underline{z}'; V, J, M)} \Big|_{J=M=0} \\
&= - \frac{\delta R_{\perp(\parallel)}(\underline{z}; J, M)}{\delta J_{\perp(\parallel)}(\underline{z})} \Big|_{J=M=0} - i R_{\perp(\parallel)}(\underline{z}) R_{\perp(\parallel)}(\underline{z}') + \overline{i R_{\perp(\parallel)}(\underline{z}; V, J, M) R_{\perp(\parallel)}(\underline{z}'; V, J, M)} \Big|_{J=M=0}.
\end{aligned} \tag{54}$$

Here, we have introduced the functionals:

$$G(\underline{x}, \underline{y}; V, J, M) = -i \frac{\delta \mathbf{W}[J, M; V]}{\delta M(\underline{y}, \underline{x})}, \quad R_{\parallel(\perp)}(\underline{z}; V, J, M) = \frac{\delta \mathbf{W}[J, M; V]}{\delta J_{\parallel(\perp)}(\underline{z})} \tag{55}$$

which depend on the random potential V . Since the average of the product $\overline{G \cdot G}$ (or $\overline{R \cdot R}$) is not the product of the averages $\overline{G} \cdot \overline{G}$ (or $\overline{R} \cdot \overline{R}$), the term $\overline{G \cdot G}$ in (53) does not cancel $\overline{G} \cdot \overline{G}$. We have already mentioned that by performing the disorder averaging of both sides of Eq. (28) we can obtain a relationship between the average photon and the average two-particle Green's functions. In other words, the average Green's functions (53) and (54) must satisfy the equation $D_{\perp} = D_{\perp}^{(0)} + D_{\perp}^{(0)} \Gamma_{\perp}^{(0)} K \Gamma_{\perp}^{(0)} D_{\perp}^{(0)}$. Obviously, from the SD equations (49) and (50) follows that the terms $\overline{G \cdot G}$, $\overline{R \cdot R}$, $\overline{G \cdot G}$ and $\overline{R \cdot R}$ in both sides of the last equation cancel each others. Thus, the cavity polaritons in the presence of a symmetry-breaking disorder manifest themselves as common poles of the term $-i \frac{\delta G(\underline{x}, \underline{y}; J, M)}{\delta M(\underline{y}', \underline{x}')} \Big|_{J=M=0}$ in (53) and $-\frac{\delta R_{\perp}(\underline{z}; J, M)}{\delta J_{\perp}(\underline{z})} \Big|_{J=M=0}$ in (54).

Next, we rewrite the mass operator Σ in the following form:

$$\begin{aligned}
\Sigma(y, x) &= -\Gamma_{\perp}^{(0)}(\underline{y}, \underline{x}' | \underline{z}) \overline{R_{\perp}(\underline{z}; V, J, M) G(\underline{x}', \underline{y}'; V, J, M)} \Big|_{J=M=0} G^{-1}(\underline{y}', \underline{x}) \\
&\quad - \Gamma_{\parallel}^{(0)}(\underline{y}, \underline{x}' | \underline{z}) K \left(\frac{\underline{x}'}{\underline{y}'} \frac{\underline{y}''}{\underline{x}''} \right) \Gamma_{\parallel}^{(0)}(\underline{y}'', \underline{x}'' | \underline{z}') D_{\parallel}^{(0)}(\underline{z}, \underline{z}') G^{-1}(\underline{y}', \underline{x}) \\
&\quad - \Gamma_{\perp}^{(0)}(\underline{y}, \underline{x}' | \underline{z}) K \left(\frac{\underline{x}'}{\underline{y}'} \frac{\underline{y}''}{\underline{x}''} \right) \Gamma_{\perp}^{(0)}(\underline{y}'', \underline{x}'' | \underline{z}') D_{\perp}^{(0)}(\underline{z}, \underline{z}') G^{-1}(\underline{y}', \underline{x}) \\
&\quad - i \Lambda K \left(\begin{array}{cc} \mathbf{r}, \underline{t}, j & \mathbf{r}, \underline{t}^I, j^I \\ \mathbf{r}^{II}, \underline{t}^{II}, j^{II} & \mathbf{r}, \underline{t}^I, j^I \end{array} \right) G_{j^{II} j'}^{-1}(\mathbf{r}^{II}, \underline{t}^{II}, \mathbf{r}', \underline{t}').
\end{aligned} \tag{56}$$

The first term in (56) is the Hartree term in the presence of a disorder. The second and the third terms link the mass operator to the average two-particle Green's function, and therefore, they are the Fock contributions to the mass operator. By introducing the vertex function $\Gamma_{\parallel(\perp)}$:

$$K^{(0)} \left(\frac{\underline{x}}{\underline{y}} \frac{\underline{y}'}{\underline{x}'} \right) \Gamma_{\parallel(\perp)}(\underline{y}', \underline{x}' | \underline{z}') D_{\parallel(\perp)}(\underline{z}', \underline{z}) = K \left(\frac{\underline{x}}{\underline{y}} \frac{\underline{y}'}{\underline{x}'} \right) \Gamma_{\parallel(\perp)}^{(0)}(\underline{y}', \underline{x}' | \underline{z}') D_{\parallel(\perp)}^{(0)}(\underline{z}', \underline{z}), \tag{57}$$

where the free two-particle propagator $K^{(0)}$ is given by:

$$K^{(0)} \left(\frac{\underline{x}}{\underline{y}} \frac{\underline{y}'}{\underline{x}'} \right) = -i G(\underline{x}, \underline{y}') G(\underline{x}', \underline{y}), \tag{58}$$

one can rewrite the mass operator in the following form:

$$\begin{aligned}
\Sigma(\underline{y}, \underline{x}) &= -\Gamma_{\perp}^{(0)}(\underline{y}, \underline{x}' | \underline{z}) \overline{R_{\perp}(\underline{z}; V, J, M) G(\underline{x}', \underline{y}'; V, J, M)} \Big|_{J=M=0} G^{-1}(\underline{y}', \underline{x}) \\
&\quad + i \Gamma_{\parallel}^{(0)}(\underline{y}, \underline{x}' | \underline{z}') G(\underline{x}', \underline{y}') \Gamma_{\parallel}(\underline{y}', \underline{x} | \underline{z}') D_{\parallel}(\underline{z}, \underline{z}') + i \Gamma_{\perp}^{(0)}(\underline{y}, \underline{x}' | \underline{z}') G(\underline{x}', \underline{y}') \Gamma_{\perp}(\underline{y}', \underline{x} | \underline{z}') D_{\perp}(\underline{z}, \underline{z}') \\
&\quad - i \Lambda K \left(\begin{array}{cc} \mathbf{r}, \underline{t}, j & \mathbf{r}, \underline{t}^I, j^I \\ \mathbf{r}^{II}, \underline{t}^{II}, j^{II} & \mathbf{r}, \underline{t}^I, j^I \end{array} \right) G_{j^{II} j'}^{-1}(\mathbf{r}^{II}, \underline{t}^{II}, \mathbf{r}', \underline{t}').
\end{aligned} \tag{59}$$

By comparing the expression for the mass operator (32) in the absence of a symmetry-breaking disorder with Eq. (56), we find two differences. The first one is the presence of an additional term, $\Sigma_{\Lambda} = -i \Lambda K G^{-1}$. The second difference is related to the corresponding Hartree

terms. In the presence of a symmetry-breaking disorder the Hartree term depends on the average of the product $\overline{R_{\perp}(\underline{z}; V, J, M) G(\underline{x}', \underline{y}'; V, J, M)}$ of two random functionals, defined by Eq. (55).

Let us for a moment replace the average $\overline{R_{\perp} G}$ with the

product of the averages $R_\perp G$:

$$\overline{R_\perp(\underline{z}; V, J, M) G(\underline{x}, \underline{y}; V, J, M)}|_{J=M=0} \rightarrow R_\perp(\underline{z}) G(\underline{x}, \underline{y}). \quad (60)$$

Although the average of the product of two random quantities is not equal to the product of their averages, the replacement (60) greatly simplifies the equations and allows us to map them to the Zittartz's equations. Note, that in his work Zittartz took into account only the lowest-order contribution from the disorder to the mass operator Σ_Λ , which corresponds to the replacement of K by the free two-particle propagator $K^{(0)}$. In this approximation we calculate for the mass operator:

$$\begin{aligned} \Sigma(\underline{y}, \underline{x}) = & -\Gamma_\perp^{(0)}(\underline{y}, \underline{x}|\underline{z}) R_\perp(\underline{z}) + \delta(\mathbf{r} - \mathbf{r}') \Lambda G_{j'j}(\mathbf{r}, \underline{t}', \mathbf{r}, \underline{t}) \\ & + i\Gamma_\parallel^{(0)}(\underline{y}, \underline{x}' | \underline{z}') G(\underline{x}', \underline{y}') \Gamma_\parallel(\underline{y}', \underline{x} | \underline{z}') D_\parallel(\underline{z}, \underline{z}'). \end{aligned} \quad (61)$$

The first and the third terms in (61) represent the photonic and excitonic contributions to the order parameter. By nullifying the sources in Eq. (49) we obtain a relationship between the average photonic field R_\perp and the average single-particle Green's function:

$$R_\perp(\underline{z}) = iD_\perp^{(0)}(\underline{z}, \underline{z}') \Gamma_\perp^{(0)}(\underline{y}, \underline{x} | \underline{z}') G(\underline{x}, \underline{y}). \quad (62)$$

The last equation leads to an equation, similar to Eq. (17):

$$R_\perp(\mathbf{k}) = \sum_{\mathbf{q}} g(\mathbf{k} - \mathbf{q}) \int \frac{d\omega}{2\pi} G_{cv}(\mathbf{q}; \omega) / (\Omega^2(\mathbf{k}) - \mu^2). \quad (63)$$

The sum of photonic and excitonic contributions to the order parameter is:

$$\Delta(\mathbf{k}) = \sum_{\mathbf{q}} \left[\frac{g(\mathbf{k} - \mathbf{q})}{\Omega^2(\mathbf{k}) - \mu^2} + V(\mathbf{k} - \mathbf{q}) \right] \int \frac{d\omega}{2\pi} G_{cv}(\mathbf{q}, \omega). \quad (64)$$

Since our random potential has a variance (33), the equation (21) of Zittartz assumes the following form:

$$\tilde{G}_{ij}(\omega) = \Lambda \sum_{\mathbf{q}} G_{ij}(\mathbf{k}, \omega). \quad (65)$$

The Fourier transform of the single-particle Green's function has the form (Eq. (24) of Zittartz):

$$\begin{aligned} \hat{G}(\mathbf{k}, \omega) = & \begin{pmatrix} G_{cc}(\mathbf{k}, \omega) & G_{cv}(\mathbf{k}, \omega) \\ G_{cv}(\mathbf{k}, \omega) & G_{vv}(\mathbf{k}, \omega) \end{pmatrix} = -\frac{1}{D} \times \\ & \begin{pmatrix} \omega - \tilde{G}_{vv}(\omega) + \eta(\mathbf{k}) + i0^+ & \Delta(\mathbf{k}) + \tilde{G}_{cv}(\omega) \\ \Delta(\mathbf{k}) + \tilde{G}_{cv}(\omega) & \omega - \tilde{G}_{cc}(\omega) - \eta(\mathbf{k}) - i0^+ \end{pmatrix}, \end{aligned} \quad (66)$$

where

$$\begin{aligned} D = & [\Delta(\mathbf{k}) + \tilde{G}_{cv}(\omega)]^2 - (\omega - \tilde{G}_{cc}(\omega) - \eta(\mathbf{k}) + i0^+) \times \\ & (\omega - \tilde{G}_{vv}(\omega) + \eta(\mathbf{k}) - i0^+). \end{aligned} \quad (67)$$

Here, $\eta(\mathbf{k})$ depends on the chemical potential, and is defined self-consistently by the solution of the following equation:

$$\begin{aligned} \eta(\mathbf{k}) = & \frac{1}{2} \left[E_g + \frac{\pi^2}{2m_{exc}L^2} + \frac{\mathbf{k}^2}{2m_{exc}} - \mu \right] \\ & - \frac{1}{2} \sum_{\mathbf{q}} V(\mathbf{k} - \mathbf{q}) \int \frac{d\omega}{2\pi} [G_{cc}(\mathbf{q}, \omega) + G_{vv}(\mathbf{q}, \omega)]. \end{aligned} \quad (68)$$

Equations (64)-(68) form a closed set of equations, that can be solved at any density. In the high density regime we can neglect the small corrections to η due to G_{cc} and G_{vv} , and map our equations (64)-(68) to the zero-temperature version of Eq. (1). As a result one could come up with the conclusion that the order parameter and the energy gap are gradually suppressed up to a critical disorder strength.

Strictly speaking, $\overline{RG} \neq RG$, and therefore, all results obtained by using approximation (60) should be considered questionable. More importantly, the \overline{RG} term does not allow us to prove the existence of the Goldstone mode below the critical temperature, as we did in Sec II. Going beyond the assumption (60) is a very challenging task, which requires to take into account diagrammatically irreducible vertex parts and an infinite number of diagrams neglected by the assumption $\overline{RG} = RG$.

IV. SUMMARY

We have applied the CPT Green's function formalism to the problem of cavity polaritons in the presence of a symmetry-breaking disorder. In contrast with the non-linear sigma-model and the saddle-point equations, the Hartree term in the mass operator does not allow us to map the SD equations to the corresponding equations in the work by Zittartz.

The saddle-point approximation and the replica trick could be responsible for the different expressions for the mass operator in the presence of a disorder. In the absence of a symmetry-breaking disorder, the saddle-point approximation leads not only to the correct gap equation in the high density regime, but by investigating the Gaussian fluctuations about the saddle point one can obtain the collective mode spectrum^{10,19} as well. Thus, we might suggest that the replica trick is not the right tool to perform the averaging over the random potential in the case of cavity polaritons in the presence of a symmetry-breaking disorder.

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